Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes) Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics FP1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Pearson

F1 Specimen.

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Answer ALL questions. Write your answers in the spaces provided.

1. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{n}{4}(n+a)(n+b)(n+c)$$

where a, b and c are integers to be found.

(4)

$\sum_{r=1}^{n} r^3 - 3r.$

$\sum r^3 - 3\sum r.$

$\frac{1}{4}n^2(n+1)^2 - \frac{3n(n+1)}{\cancel{2}}$

$\frac{n}{4}(n+1)\left[n(n+1) - 6\right].$

$= \frac{n}{4}(n+1)(n^2+n-6)$

$\frac{n}{4}(n+1)(n+3)(n-2)$

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2. A parabola P has cartesian equation $y^2 = 28x$. The point S is the focus of the parabola P.

(a) Write down the coordinates of the point S.

(1)

Points A and B lie on the parabola P. The line AB is parallel to the directrix of P and cuts the x-axis at the midpoint of OS, where O is the origin.

(b) Find the exact area of triangle ABS.

(4)

(a) $28 = 4a$

$a = 7.$

$(7, 0) \Rightarrow S.$

(b) MP of OS.

$(0,0)$ $(7,0)$

$= (3.5, 0)$

$\therefore$ x-co-ordinate of A and B = 3.5.

$y^2 = 28(3.5)$

$y = \pm 7\sqrt{2}$

$$\frac{1}{2}\left| \begin{matrix} 3.5 \\ 7\sqrt{2} \end{matrix} \times \begin{matrix} 3.5 \\ -7\sqrt{2} \end{matrix} \times \begin{matrix} 7 \\ 0 \end{matrix} \times \begin{matrix} 3.5 \\ 7\sqrt{2} \end{matrix} \right|$$

$$\frac{1}{2}\left| \left(\frac{-49\sqrt{2}}{2} + 49\sqrt{2}\right) - \left(\frac{49\sqrt{2}}{2} - 49\sqrt{2}\right) \right|$$

$$\frac{1}{2}\left| \frac{49\sqrt{2}}{2} - - \frac{49\sqrt{2}}{2} \right|$$

$$= \frac{49\sqrt{2}}{2}$$

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3.

$$f(x) = x^2 + \frac{3}{x} - 1, \qquad x < 0$$

The only real root, α, of the equation $f(x) = 0$ lies in the interval $[-2, -1]$.

(a) Taking -1.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.
(5)

(b) Show that your answer to part (a) gives α correct to 2 decimal places.
(2)

(a) $f(x) = x^2 + \frac{3}{x} - 1$

$f(-1.5) = (-1.5)^2 + \frac{3}{-1.5} - 1$

$= -\frac{3}{4}$

$f'(-1.5)$

$f'(x) = 2x - \frac{3}{x^2}$

$= 2(-1.5) - \frac{3}{(-1.5)^2}$

$= -\frac{13}{3}$

$= -1.5 - \left[\frac{-0.75}{-13/3}\right]$

$= \underline{\underline{-1.67}}\ (2dp)$

(b) $f(-1.675) = 0.0146$

$f(-1.665) = -0.0296$.

sign change

$\therefore$ ~~α~~ α is correct to 2dp

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4. Given that

$$\mathbf{A} = \begin{pmatrix} k & 3 \\ -1 & k+2 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) show that $\det(\mathbf{A}) > 0$ for all real values of k,

(3)

(b) find $\mathbf{A}^{-1}$ in terms of k.

(2)

Det A.

$k(k+2) - (-3)$

$= k^2 + 2k + 3.$

$= (k+1)^2 + 2$

$\therefore (k+1)^2 + 2 > 0$ for all real values of k.

(b) $\dfrac{1}{k^2+2k+3}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$

Leave blank

5.

$$2z + z^* = \frac{3 + 4i}{7 + i}$$

Find z, giving your answer in the form $a + bi$, where a and b are real constants. You must show all your working.

(5)

$z = a + bi.$

$2(a+bi) + (a-bi) = \frac{3+4i}{7+i}$

$\frac{3+4i}{(7+i)} \frac{(7-i)}{(7-i)}$

$= 21 - 3i + 28i - 4i^2$

$= \frac{25+25i}{49+1} = \frac{25+25i}{50}$

$= \frac{1}{2} + \frac{1}{2}i$

$2a + 2bi + a - bi.$

$3a + bi = \frac{1}{2} + \frac{1}{2}i.$

$3a = \frac{1}{2}$

$a = \frac{1}{6}$

$b = \frac{1}{2}$

$\therefore z = \frac{1}{6} + \frac{1}{2}i$

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6. The rectangular hyperbola H has equation $xy = 25$

(a) Verify that, for $t \neq 0$, the point $P\left(5t, \dfrac{5}{t}\right)$ is a general point on H.

(1)

The point A on H has parameter $t = \dfrac{1}{2}$

(b) Show that the normal to H at the point A has equation

$$8y - 2x - 75 = 0$$

(5)

This normal at A meets H again at the point B.

(c) Find the coordinates of B.

(4)

(a) $xy = 25$

$5\cancel{t} \times \dfrac{5}{\cancel{t}} = 25$

$\therefore P\left(5t, \dfrac{5}{t}\right)$ is the general point.

(b) when $t = 1/2$ $(5/2, 10)$

$y = \dfrac{25}{x}$

$\dfrac{dy}{dx} = \dfrac{-25}{x^2}$ at $x = 5/2$

$\dfrac{dy}{dx} = -4$

$\therefore$ at normal grad $= \dfrac{1}{4}$

$y - 10 = \dfrac{1}{4}\left(x - \dfrac{5}{2}\right)$

$y = \dfrac{1}{4}x - \dfrac{5}{8} + 10$

$y = \dfrac{1}{4}x + \dfrac{75}{8}$

$8y - 2x - 75 = 0$ as req.

(c) $\dfrac{1}{4}x + \dfrac{75}{8} = \dfrac{25}{x}$

$\dfrac{x^2}{4} + \dfrac{75}{8}x - 25 = 0$

$$\frac{-\frac{75}{8} \pm \sqrt{\left(\frac{75}{8}\right)^2 - 4\left(\frac{1}{4} \times -25\right)}}{2 \times 1/4}$$

$x = -40$ or $2.5 \leftarrow P$

$\therefore$ when $x = -40$ $y = -\dfrac{5}{8}$

$B = (-40, -5/8)$

Leave blank

7.

$$\mathbf{P} = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **P**.
(3)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line with equation $y = x$

(b) Write down the matrix **Q**.
(1)

Given that the transformation V followed by the transformation U is the transformation T, which is represented by the matrix **R**,

(c) find the matrix **R**.
(2)

(d) Show that there is a value of k for which the transformation T maps each point on the straight line $y = kx$ onto itself, and state the value of k.
(4)

(a) $\cos^{-1}\left(\frac{5}{13}\right) = 67.3°$

$= 67°$ Rotation anticlockwise centre $(0,0)$

(b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(c) $UV = T$.

$\begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$R = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}$

(d) since all co-ordinates are mapped on that line

$\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}\begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix}$

$\begin{pmatrix} -\frac{12}{13}x + \frac{5kx}{13} \\ \frac{5x}{13} + \frac{12kx}{13} \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix}$

$-\frac{12x}{13} + \frac{5kx}{13} = x$

$\underline{k = 5}$

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8.

$$f(z) = z^4 + 6z^3 + 76z^2 + az + b$$

where a and b are real constants.

Given that $-3 + 8i$ is a complex root of the equation $f(z) = 0$

(a) write down another complex root of this equation. **(1)**

(b) Hence, or otherwise, find the other roots of the equation $f(z) = 0$ **(6)**

(c) Show on a single Argand diagram all four roots of the equation $f(z) = 0$ **(2)**

(a) $-3-8i$

(b) sum of roots.

$-3-8i + -3 + 8i$

$= \underline{\underline{-6}}$

Product of roots.

$(-3-8i)(-3+8i)$

$= 9 - 64i^2 = 73.$

$(z^2+6z+73)(az^2+bz+c)$

$az^4 = 1z^4 \quad \underline{a=1}$

$\cancel{73az^2}$

$6az^3 + bz^3 = 6z^3.$

$6+b=6.$

$\underline{b=0}$

$cz^2 + 73az^2 = 76.$

$c + 73 = 76$

$\underline{c=3}$

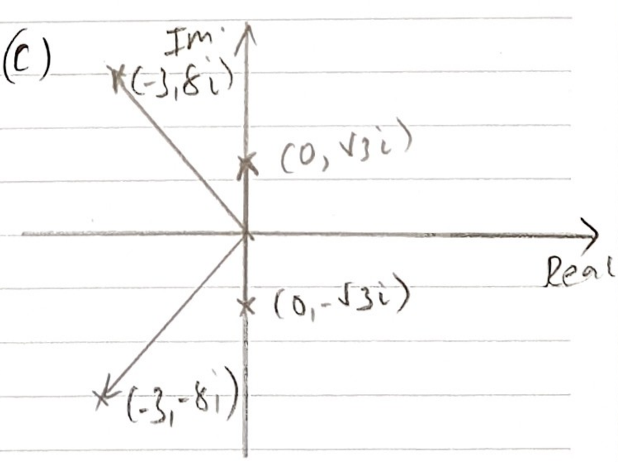
$\therefore$ other quadratic factor =

(z^2+3)

$z^2 = -3.$

$z = \underline{\underline{\pm\sqrt{3}i}}$

(c)



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9. The quadratic equation

$$2x^2 + 4x - 3 = 0$$

has roots α and β.

Without solving the quadratic equation,

(a) find the exact value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(5)

(b) Find a quadratic equation which has roots $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(4)

(a) $\alpha + \beta = \frac{-4}{2} = -2$.

$\alpha\beta = \frac{-3}{2}$.

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 4 - 2\left(-\frac{3}{2}\right)$

$= 7$

$\alpha^3 + \beta^3$

$(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= (-2)^3 - 3(-1.5)(-2)$

$= \underline{\underline{-17}}$

(b) Sum of roots.

$\alpha^2 + \beta^2 + \alpha + \beta$

$= 7 + -2 = \underline{\underline{5}}$

Product of roots.

$\alpha^2\beta^2 + \alpha^3 + \beta^3 + \alpha\beta$.

$(\alpha\beta)^2 + (\alpha^3 + \beta^3) + \alpha\beta$.

$= \left(\frac{-3}{2}\right)^2 + (-17) + (-3/2)$

$= -\frac{65}{4}$

$x^2 - 5x - \frac{65}{4} = 0$.

$4x^2 - 20x - 65 = 0$

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10. (i) A sequence of positive numbers is defined by

$$u_1 = 5$$
$$u_{n+1} = 3u_n + 2, \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 2 \times (3)^n - 1$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3+2n)}{3^n}$$

(6)

(i) Basis

let $n=1$

$u_1 = 2 \times (3)^1 - 1$
$= 6 - 1 = 5$ (✓)

Assumption

let's assume the following statement is true for $n=k$.

$u_k = 2 \times (3)^k - 1$

Induction

let $n = k+1$

$u_{k+1} = 3u_k + 2$.

From the previous stage

$u_k = 2 \times (3)^k - 1$

$= 3(2 \times (3)^k - 1) + 2$.

$= 2 \times (3)^{k+1} - 3 + 2$

$= 2 \times (3)^{k+1} - 1$

$\therefore$ true for $n=k+1$

Conclusion

If true for $n=k$, then proved true for $n=k+1$. Since true for $n=1$, $\therefore$ true for $n \geq 1$.

(ii) Basis

let $n=1$.

$\sum_{r=1}^{1} \frac{4(1)}{3^1} = \frac{4}{3}$. (LHS)

RHS)

$3 - \frac{(3+2)}{3^1} = 3 - \frac{5}{3} = \frac{4}{3}$.

$\therefore$ true for $n=1$.

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Question 10 continued

Assumption

Let $n=k$, and assume the following statement is true for $n=k$.

$$\sum_{r=1}^{k} \frac{4k}{3^k} = 3 - \frac{(3+2k)}{3^k}.$$

Induction

Let $n=k+1$.

$$\sum_{r=1}^{k+1} = \sum_{r=1}^{k} + \frac{4(k+1)}{3^{k+1}} \cancel{3 - \frac{(3+2(k+1))}{3^{k+1}}}$$

$$= \cancel{\frac{4k}{3^k}}$$

$$= 3 - \frac{3+2k}{3^k} + \frac{4(k+1)}{3^{k+1}}$$

$$3 - \left[\frac{3(3+2k)+4(k+1)}{3^{k+1}}\right]$$

$$= 3 - \left(\frac{5+2k}{3^{k+1}}\right)$$

$$= 3 - \frac{(3+2(k+1))}{3^{k+1}}$$

$\therefore$ true for $n=k+1$.

Conclusion

If true for $n=k$, then proved true for $n=k+1$. Since true for $n=1$ $\therefore$ true for

$n \in \mathbb{Z}^+$